

The Rubakov-Callan Scattering on the Supergravity Monopole

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We study small perturbations around the supersymmetric CVMN monopole solution of the gauged supergravity in D=4. We find that the perturbation spectrum contains an infinite tower of Coulomb-type bound states both in the bosonic and fermionic parts of the supergravity multiplet. Due to supersymmetry, the eigenvalues are the same for the two bosonic parity sectors, as well as for the fermionic sector. We also find that the fermion scattering on the monopole is accompanied by isospin flip. This is analogous to the Rubakov-Callan effect of monopole catalysis of proton decay and suggests that there could be a similar effect of catalysis for decay of fermionic systems in supergravity.

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Introduction.— The Rubakov-Callan effect [1] can be viewed as a consequence of the isospin flip for Dirac fermions interacting with the t' Hooft-Polyakov magnetic monopole [2]. When scattered on the monopole, fermions change their quantum numbers, so that outgoing particles are not the same as incoming ones. When interacting with systems of bound fermions, as for example quarks inside a proton, the monopole changes the quark colors, thus rendering the system unstable. Although not observed in nature so far, such a monopole catalysis of proton decay is very interesting theoretically.

There are other interesting theoretical effects for magnetic monopoles (see [3] for a review). For example, scattering of even parity Yang-Mills and Higgs quanta can resonantly excite the monopole, giving rise to a long living breathing state [4], while the odd parity quanta can be trapped by the monopole to form bound states [5]. The monopole can also confine zero energy fermions, in agreement with the index theorem [6].

In this letter we study analogous effects, but in connection to the supergravity (SUGRA) monopole solution of Chamseddine-Volkov-Maldacena-Nunez (CVMN). This is an exact solution [7] of equations of gauged SUGRA in D=4 that preserves four supersymmetries (SUSY) and contains a Yang-Mills field whose structure is exactly the same as for the t' Hooft-Polyakov monopole. This solution can be promoted to D=10 as a string theory vacuum [7], in which case it can be interpreted as a holographic dual of the N=1 super-Yang-Mills [8].

In what follows, we maintain the original interpretation of the solution as magnetic monopole in D=4 and study its small excitations within the SUGRA multiplet. We find that boson fluctuations split into two parity sectors

and admit an infinite tower of bound states with the same eigenvalues for both parities. There are also bound states with exactly the same eigenvalues in the fermion sector too. We then study the fermion scattering and observe the isospin flip phenomenon similar to the one discussed by Rubakov and Callan, even though we do not consider Dirac fermions but interacting spin-3/2 and spin-1/2 Majorana fields. This suggests that there could be a similar effect of monopole catalysis of bound fermionic systems in SUGRA.

SUGRA bosons.— The $N = 4$ gauged $SU(2) \times SU(2)$ supergravity of Freedman and Schwarz (FS) [9] contains the gravitational field $g_{\mu\nu}$, the axion \mathbf{a} , dilaton Φ , and two non-Abelian gauge fields A_μ^a and B_μ^a ($a = 1, 2, 3$) with gauge couplings e_A and e_B , as well as the fermions. One can consistently truncate the theory to the sector where $B_\mu^a = e_B = 0$, after which one can rescale the remaining fields to achieve the condition $e_A = 1$. The action density of the theory then reads

$$\mathcal{L} = -\frac{1}{4} R + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2} e^{-4\Phi} \partial_\mu \mathbf{a} \partial^\mu \mathbf{a} - \frac{1}{4} e^{2\Phi} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\mathbf{a}}{2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + \frac{1}{8} e^{-2\Phi} + \text{fermions}, \quad (1)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon_{abc} A_\mu^b A_\nu^c$ is the gauge field tensor and $\tilde{F}_{\mu\nu}^a$ its dual. One can consistently set the fermionic fields to zero and study the purely bosonic fields. Assuming the latter to be spherically symmetric, the most general line element can be parameterized in spherical coordinates t, r, ϑ, φ as

$$ds^2 = 2e^{2\Phi} (e^{2\nu} dt^2 - e^{2\lambda} dr^2 - U^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)). \quad (2)$$

The most general SO(3) invariant gauge field is [10]

$$\mathbf{T}_a A_\mu^a dx^\mu = (\Omega_t dt + \Omega_r dr) \mathbf{T}_3 + (k_2 \mathbf{T}_1 + k_1 \mathbf{T}_2) d\vartheta + (k_2 \mathbf{T}_2 - k_1 \mathbf{T}_1) \sin \vartheta d\varphi + \mathbf{T}_3 \cos \vartheta d\varphi \quad (3)$$

with $[\mathbf{T}_a, \mathbf{T}_b] = i\epsilon_{abc}\mathbf{T}_c$ being the $SU(2)$ generators. Here $\Phi, \nu, \lambda, U, \Omega_t, \Omega_r, k_1, k_2$ depend on t, r . In the static case one can set $\nu = \lambda = \mathbf{a} = \Omega_t = \Omega_r = k_2 = 0$ and $\Phi = \phi(r)$, $k_1 = w(r)$, $U = Y(r)$. The field equations then admit an exact solution describing the CVMN monopole [7],

$$w = \frac{r}{\sinh r}, \quad e^{2\phi} = a \frac{\sinh r}{Y}, \quad Y^2 = 2r \coth r - w^2 - 1, \quad (4)$$

where a as an integration constant. We study time-dependent perturbations of this solution within the ansatz (2),(3), so that to set

$$\begin{aligned} \Phi &= \phi + \delta\phi, & k_1 &= w + \delta w, & \lambda &= \delta\lambda, & \nu &= \delta\nu, \\ \omega_1 &= \delta\omega_1, & k_2 &= \delta k_2, & \mathbf{a} &= \delta\mathbf{a}, \end{aligned} \quad (5)$$

where the perturbations depend on t, r and are assumed to be small. The line element (2) has a residual symmetry $t \rightarrow t + g(t, r)$, $r \rightarrow r + f(t, r)$ with $\partial_r g = \partial_t f$, while the ansatz (3) is invariant under $\Omega_a \rightarrow \Omega_a + \partial_a \alpha(t, r)$, $k_1 + ik_2 \rightarrow e^{i\alpha}(k_1 + ik_2)$ ($a = t, r$). These symmetries can be used to impose the gauge conditions $\Omega_t = 0$ and $U = Y(r)$.

Inserting (5) into the field equations for the action (1) and linearizing with respect to perturbations, the perturbation equations split into two independent parity groups. In what follows we shall only outline the results of the complicated detailed calculations which will appear elsewhere. Let us consider first the even parity group containing $\delta\phi, \delta w, \delta\nu, \delta\lambda$. The key observation is that the linearized tr component of Einstein equations is a total derivative with respect to time, which after integrating gives an algebraic relation expressing $\delta\lambda$ in terms of $\delta\phi$ and δw . The rr component of Einstein equations then can be resolved with respect to $\delta\nu$, so that the linearized Yang-Mills and dilaton equations will contain only $\delta\phi$ and δw . With $\delta w = e^{i\omega t} e^{-\phi} B_1(r)$ and $\delta\phi = e^{i\omega t} e^{-\phi} B_2(r)/Y$, these equations reduce to a two-channel Schrödinger system

$$\begin{aligned} -B_1'' - CB_1' + V_{11}B_1 + V_{12}B_2 &= \omega^2 B_1, \\ -B_2'' + CB_2' + V_{22}B_2 + V_{21}B_1 &= \omega^2 B_2, \end{aligned} \quad (6)$$

where $' \equiv \frac{d}{dr}$ and C, V_{ik} are real functions of ϕ, w, Y with $V_{21} = V_{12} + C'$.

Equations in the odd parity sector contain $\delta\mathbf{a}, \delta\Omega_r$ and δk_2 . Setting $\delta\mathbf{a} = \omega \cos(\omega t) e^\phi B_1/Y$ and $\delta\Omega_r = 2 \sin(\omega t) e^{-\phi} (wYB_2 + (w^2 - 1)B_1)/Y^3$ and also $\delta k_2 = \sin(\omega t) e^{-\phi} (e^{-\phi} (we^\phi B_2)'/w + 2w'B_1/Y)$, these equations also assume the Schrödinger form (6), but the potential V_{ik} is not the same as in the even-parity case. However, the behavior at $r \rightarrow \infty$ is similar for both parities, since $C \sim V_{12} \sim V_{21} \sim e^{-r} \rightarrow 0$, so that the system (6) diagonalizes, while

$$V_{11} \sim V_{22} \sim \frac{1}{4} - \frac{3}{4r} + O(r^{-2}). \quad (7)$$

Since $V_{11}(\infty) = V_{22}(\infty)$, it follows that, unlike for the t' Hooft-Polyakov monopole [4], the SUGRA monopole

n	$\frac{1}{4} - \frac{9}{64n^2}$	even ⁺	even ⁻
1	0.10937	0.101710	0.201961
2	0.21484	0.217134	0.230873
3	0.23437	0.235150	0.239792
4	0.24121	0.241552	0.243665
5	0.24437	0.244553	0.245689

Table I. Bound state eigenvalues for Eqs.(6).

does not have resonant excitations. The spectrum contains scattering states with $\omega^2 > 1/4$, while for $\omega^2 < 1/4$ there should be an infinite tower of bound states – since the potential contains the attractive Coulombian tail. In the idealized case, if we had exactly $V_{11} = V_{22} = 1/4 - 3/(4r)$ and $V_{12} = V_{21} = C = 0$, the bound states eigenvalues would be $\omega_n^2 = 1/4 - (3/(4n))^2$ with $n = 1, 2, \dots$ and for every eigenvalue there would be two different solutions.

Solving Eqs.(6) numerically, we indeed find bound states with $\omega^2 \approx \omega_n^2$. The double degeneracy present in the idealized case is lifted for the full system, and for a given n we find two different solutions with slightly different eigenvalues that we call n^+ and n^- , since one of them has $B_1 \approx B_2$ and the other $B_1 \approx -B_2$. The first ten eigenvalues are shown in Table 1, where it could be seen that when n increases, they indeed approach the Coulombian values ω_n^2 . We notice, however, something unusual, since for both parity values the eigenvalues turn out to be the same, at least up to six decimal places as shown in the table, even though the potentials V_{ik} are different. The explanation of this remarkable coincidence is due to supersymmetry. We next study the fermionic sector.

SUGRA fermions.– The fermions in the FS model are the gravitino ψ_μ and gaugino χ . These are Majorana spinors endowed with an isospin index, so that they have altogether 80 complex components. Neglecting their self-interactions, their equations of motion read [9],[11]

$$\begin{aligned} \mathcal{R}^\lambda &\equiv \varepsilon^{\lambda\mu\nu\rho} \gamma_5 \gamma_\mu \hat{D}_\nu \psi_\rho - \frac{1}{\sqrt{2}} e^{-\phi} \sigma^{\lambda\nu} \psi_\nu \\ &+ \left(\frac{i}{2} e^\phi \mathcal{F} - \frac{1}{\sqrt{2}} \gamma^\nu \partial_\nu \phi - \frac{i}{4} e^{-\phi} \right) \gamma^\lambda \chi = 0, \\ \mathcal{P} &\equiv i\gamma^\mu D_\mu \chi + \gamma^\mu \left(\frac{1}{\sqrt{2}} \gamma^\rho \partial_\rho \phi + \frac{i}{2} e^\phi \mathcal{F} - \frac{i}{4} e^{-\phi} \right) \psi_\mu = 0, \end{aligned} \quad (8)$$

where $D_\mu = \partial_\mu + \frac{1}{4} \omega_\mu^{\alpha\beta} \gamma_\alpha \gamma_\beta - \mathbf{T}^a A_\mu^a$ with $\omega_\mu^{\alpha\beta}$ being the spin connection and $\hat{D}_\mu = D_\mu - \frac{i}{2\sqrt{2}} e^\phi \mathcal{F} \gamma_\mu$ with $\mathcal{F} = \mathbf{T}^a F_{\alpha\beta}^a \gamma^\alpha \gamma^\beta$. These equations are invariant under the SUSY transformations $\psi_\mu \rightarrow \psi_\mu + \delta_\epsilon \psi_\mu$, $\chi \rightarrow \chi + \delta_\epsilon \chi$ with

$$\begin{aligned} \delta_\epsilon \psi_\mu &= \left(D_\mu + \frac{i}{2\sqrt{2}} e^\phi \mathcal{F} \gamma_\mu - \frac{i}{4\sqrt{2}} e^{-\phi} \gamma_\mu \right) \epsilon, \\ \delta_\epsilon \chi &= \left(\frac{i}{\sqrt{2}} \gamma^\mu \partial_\mu \phi - \frac{1}{2} e^\phi \mathcal{F} + \frac{1}{4} e^{-\phi} \right) \epsilon, \end{aligned} \quad (9)$$

where ϵ is the spinor SUSY parameter. Due to this invariance, there exist identity relations between the equations (Bianchi identities)

$$D_\rho \mathcal{R}^\rho + \frac{i}{2\sqrt{2}} \gamma_\rho (e^\phi \mathcal{F} - \frac{1}{2} e^{-\phi}) \mathcal{R}^\rho - (\frac{i}{\sqrt{2}} \gamma^\rho \partial_\rho \phi + \frac{1}{2} e^\phi \mathcal{F} - \frac{1}{4} e^{-\phi}) \mathcal{P} = 0. \quad (10)$$

It is worth noting that the fermion equations are SUSY invariant iff the boson background is on-shell. The monopole background (4) is on-shell and moreover it is supersymmetric, since it admits four non-trivial spinors ϵ_0 such that $\delta_{\epsilon_0} \psi_\mu = \delta_{\epsilon_0} \chi = 0$, so that it is invariant under SUSY transformations generated by ϵ_0 [7].

Let us collectively denote the bosons by B and fermions by F . Their SUSY variations can be schematically expressed as $\delta_\epsilon B = \bar{\epsilon} F$ and $\delta_\epsilon F = D(B) \epsilon$ where $D(B)$ is a covariant derivative operator. Let us set $F = 0$ and $B = B_0 + \delta B$ where B_0 is the monopole and δB its perturbation. Since this configuration is on-shell, so will be its SUSY variations. Let us consider the variations induced by the Killing spinors ϵ_0 . One has $\delta_{\epsilon_0} B = 0$, while

$$\delta_{\epsilon_0} F = D(B_0 + \delta B) \epsilon_0 \approx D(B_0) \epsilon_0 + D(\delta B) \epsilon_0 = D(\delta B) \epsilon_0, \quad (11)$$

where we used the fact the the background is SUSY-invariant i.e. $D(B_0) \epsilon_0 = 0$. By construction, (11) should fulfill the fermion equations. Therefore, to every perturbative solution δB in the bosonic sector there corresponds a solution $D(\delta B) \epsilon_0$ in the fermionic sector. More precisely, it is given by (9) with $\epsilon = \epsilon_0$.

It then follows that the fermionic equations (8) should contain all solutions of the bosonic equations. In particular, they should have the same bound state spectrum. To verify this, we should solve the system of 80 spinor equations (8). As a first step, we impose the symmetry condition on spinors, whose total angular momentum $J = L + S + I$ consists of the orbital part L , spin S and isospin I . Since $I = 1/2$, both for $S = 3/2$ (gravitino) and $S = 1/2$ (gaugino), there are integer values of L giving $J = 0$. Therefore, both the gravitino and gaugino could form spherically symmetric states. When we restrict to $J = 0$, the dependence on the angles ϑ, φ separates, and the fermionic system (8) reduces to 32 equations for 32 complex functions of t and r . In addition, the Majorana condition eliminates half of the degrees of freedom, so that we are left with only 16 equations for 16 complex amplitudes.

As a consistency check, we verify that these equations admit (9) as solutions (provided that ϵ also has $J = 0$) and that they fulfill the Bianchi identities (10). Next, we verify that if δB is a solution of the Schrödinger problem (6), either for even or for odd parity, then $D_\mu(\delta B) \epsilon_0$ fulfills the fermion equations. After this we are confident that our equations are correct, and so we proceed to solve them. In order to fix the gauge, we impose the condition

$$\gamma^0 \psi_0 + \frac{i}{\sqrt{2}} \chi = 0, \quad (12)$$

which removes all time-dependent pure gauge modes and eliminates 4 complex amplitudes out of 16 yielding 16 equations for 12 functions. It turns out that 4 of the 16 equations are algebraic and can be used to express 4 functions in terms of the other 8. As a result, there remain 12 equations for 8 functions. Assuming the harmonic time dependence $e^{i\omega t}$ for the spinors, the time variable separates. Taking linear combinations, we find that only 8 of the remaining 12 equations are differential while the remaining 4 are algebraic constraints. We check then that differentiating these constraints and using the 8 differential equations to eliminate the derivatives do not lead to new constraints. We can therefore resolve the constraints to express four amplitudes in terms of the other four, so that everything reduces to just four first order differential equations with real coefficients. Converting them to two second order equations, we finally obtain

$$\begin{aligned} -F_1'' + (U_{11} - \omega^2) F_1 + \omega U_{12} F_2 &= 0, \\ -F_2'' + (U_{22} - \omega^2) F_2 + \omega U_{21} F_1 &= 0, \end{aligned} \quad (13)$$

where U_{ik} are real functions of the background amplitudes w, ϕ, Y . Summarizing, we managed to reduce the 80 fermion equations (8) to two second order equations (13). These equations are solved to determine $F_1(r), F_2(r)$, in terms of which all components of ψ_μ, χ can then be expressed.

Before proceeding, we analyze the asymptotic behavior of solutions and find that for $r \rightarrow \infty$ one has $F_1 \sim F_2 \sim e^{-3\phi/2 - kr}$ with $k = \sqrt{1 - \omega^2/4}$. Since $\phi \sim r/2$ at large r , it follows that the solutions are always exponentially suppressed at infinity, irrespective of the value of ω . Fermions are therefore always localized around the monopole and cannot escape to infinity, as if they had no scattering states. However, this seems to be a purely kinematical effect, since passing to the string frame $ds^2 \rightarrow e^{-2\phi} ds^2$ changes the spinors as $F \rightarrow e^{3\phi/2} F$ so that they oscillate at infinity for $\omega^2 > 1/4$ and are exponentially suppressed for $\omega^2 < 1/4$. We then solve equations (13) numerically looking for bound states with $\omega^2 < 1/4$ and obtain exactly the same eigenvalues as those given in Tab.1. This result is of course natural, since we know that the fermion equations contain all solutions of the bosonic sector due to the map $\delta B \rightarrow D(\delta B) \epsilon_0$.

In the bosonic sector there are two different sets of equations, one for even parity and one for odd parity. The eigenvalues are the same in both cases, so that every eigenvalue is doubly degenerate. In the fermionic sector we obtain only one set of equations, and for every eigenvalue we find only one solution. This solution should therefore be the image of both even parity and odd parity bosonic modes. When explicitly calculating F_1, F_2 corresponding to $D(\delta B) \epsilon_0$, we obtain the same result regardless of whether δB has even or odd parity,

$$B_1^{\text{even}}, B_2^{\text{even}} \rightarrow F_1, F_2 \leftarrow B_1^{\text{odd}}, B_2^{\text{odd}}. \quad (14)$$

Therefore, the even parity and odd parity bosonic sectors can be related to each other by a change of variables

via the fermion sector. This finally explains why the spectrum is the same for both parities.

Fermion scattering.— We are now ready to analyze the scattering problem. Let us consider a wave ingoing from infinity in the F_1 channel of system (13). It will approach the monopole core, where it will excite the amplitude also in the F_2 channel. The wave reflected from the center will go back to infinity being distributed between both channels, so that for $r \rightarrow \infty$ one will have

$$F_1(r) \rightarrow e^{ikr} + a_1 e^{-ikr}, \quad F_2(r) \rightarrow a_2 e^{-ikr}, \quad (15)$$

where a_1, a_2 are complex functions of ω . Solving equa-

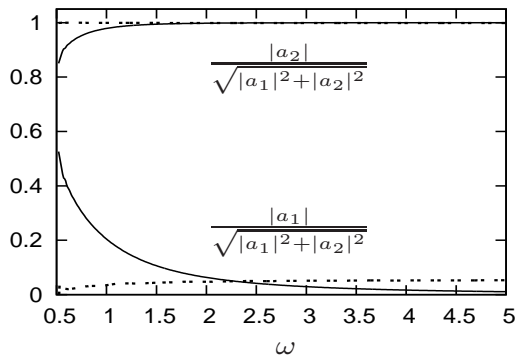


Figure 1. The reflection and transmission coefficients a_1, a_2 against ω if the incident wave is in the F_1 channel (solid lines), or in the F_2 channel (dotted lines).

tions (13) with such boundary conditions shows that the reflection coefficient $a_1(\omega)$ rapidly approaches zero

(see Fig.1). Therefore, the F_1 ingoing wave gets converted into the F_2 outgoing wave when scattered on the monopole. We call this phenomenon isospin flip, since it is very similar to the isospin flip for the Dirac fermions scattered on the t' Hooft-Polyakov monopole. The only difference is that in the latter case the fermions do not encounter a centrifugal barrier [1], whereas our equations (13) turn out to contain the $2/r^2$ centrifugal term at small r . As a result, we do not always have a 100% isospin flip, but a flip rapidly approaching 100% as the energy increases.

If the wave incident from infinity is in the F_2 channel, then the asymptotic behavior for $r \rightarrow \infty$ is given by (15) with F_1 and F_2 interchanged. As seen in Fig.1, in this case $a_1(\omega)$ does not tend to zero when ω increases, but it is always very small and approaches ≈ 0.05 , so that the isospin flip is $\approx 95\%$.

A consequence of the isospin flip on the t' Hooft-Polyakov monopole is the Rubakov-Callan effect of monopole catalysis of proton decay [1]. It is therefore suggestive that the CVMN monopole could similarly catalyze the decay of fermionic systems in SUGRA.

One can also consider the fermion zero modes. For example, differentiating the background (4) with respect to the scale parameter a gives a boson zero mode δB , which can be converted to the fermion mode via $\delta B \rightarrow D(\delta B)\epsilon_0$. However, since our spinors are Majorana and not Weyl, the relation between fermion zero modes and the monopole topology is less clear than in the standard case [6]. These and other issues will be explored in a detailed publication.

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